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Particle Swarm Optimization for Vehicle Routing Problem with Fleet Heterogeneous and Simultaneous Collection and Delivery

Bárbara de Cássia Xavier Cassins Aguiar

Federal University of Paraná - DEGRAF, Jardim das Américas
Cx Postal 19081, Curitiba- Paraná, 81531-980, Brazil

Paulo Henrique Siqueira

Federal University of Paraná - DEGRAF, Jardim das Américas
Cx Postal 19081, Curitiba- Paraná, 81531-980, Brazil

Giancarlo de França Aguiar

Federal Institute of Paraná, Rua João Negrão
Cx Postal 1285, Curitiba- Paraná, 80230-150, Brazil

Luzia Vidal de Souza

Federal University of Paraná - DEGRAF, Jardim das Américas
Cx Postal 19081, Curitiba- Paraná, 81531-980, Brazil

Abstract

This paper presents a new strategy for solving routing problems capacitated vehicles with heterogeneous fixed fleet and delivery and simultaneous collection of the application of meta-heuristic Particle Swarm Optimization Discrete

Adapted (PSODA) proposed. Particle Swarm Optimization (PSO) is a meta-heuristic developed in 1995 by Kennedy and Eberhart, based on the analysis of intelligent behavior of flocks of birds. The proposed model is applied to the Problem of School Transportation in thirty-two counties of the State of Paraná. The proposed algorithm differs from PSO classic, both in the coding of the particles as in established operations. The computational experiment conducted showed that the proposed algorithm could obtain good quality solutions. Initial results point to the effectiveness of the proposed approach for sets of analyzed problems.

Keywords: Optimization, Particle Swarm, School Transportation.

1 Introduction

Solutions to optimization problems are important in everyday life. Many social, economic, scientific and engineering problems have variables that can be adjusted to produce better results and may increase profits or reduce costs. Over the years, several research that led to the development of new techniques for solving such problems have been made.

The accelerated development of optimization techniques and its flexibility for application to complex problems are factors that favor its use. In this context, analysis, design and research involving optimization algorithms are important topics to be studied. One of the areas in which optimization techniques have been applied, and that is the objective of this research are those that involve routing problems.

The vehicle routing problem is to define roadmaps vehicles that minimize the total cost of attendance, each of which starting and ending at the depot or base of the vehicle, ensuring that each point is visited exactly once and the demand on any route does not exceed the capacity of the vehicle that meets [6] .

Among the problems of routing vehicle problem is the School Transport. In this, the vehicles collect students in their breakpoints and deliver them to their respective schools. In this paper, the problem of school transport studied can be classified as a capable routing problem with heterogeneous fleet and fixed delivery and simultaneous collection.

This paper presents a strategy for solving the Problem of School Transportation and its applications in some municipalities of Paraná. A study on the optimization algorithm for Swarm Particle (PSO) is presented, and the proposed algorithm is applied to the Vehicle Routing Problem in School Transportation.

2 **Particle Swarm Optimization**

The optimization technique Particle Swarm Optimization (PSO) was proposed by [2] like other meta- heuristics developed recently, was inspired by the social behavior found in populations. The technique was developed from the observation of flocks of birds and schools of fish in search of food in a particular region. When analyzing these groups, it is observed that their behavior is influenced by individual experience accumulated by each individual as well as the result of experience accumulated by the group. The population on the PSO called cloud (or cluster) is composed of particles that are candidate solutions for the problem.

The population of PSO called cloud (or cluster) is composed of particles that are candidate solution to the problem. The particles move in a n -dimensional space within a search subspace S [10].

Each particle p in a given iteration t has a position R^n , $\vec{X}(t)$ and a speed of displacement in space, $\vec{V}(t)$. It also has a memory containing its best position ever achieved, $p\vec{Best}(t)$ and the best position ever achieved by neighboring particles p , $g\vec{Best}(t)$. Importantly, $\vec{X}(t)$, $\vec{V}(t)$, $p\vec{Best}(t)$ and $g\vec{Best}(t)$ are n-dimensional vectors, n being determined by the problem being solved by the algorithm. If this were, for example, the well-known traveling salesman problem, n be the total number of cities to be visited as presented in [11].

Let $\vec{X}_i(t) = \{x_{i,1}(t),...,x_{i,n}(t)\}$ and $\vec{V}_i(t) = \{v_{i,1}(t),...,v_{i,n}(t)\}$ respectively, the position (vector own candidate to the solution) and speed (its rate of change) of particle i at time t, in a search space of n-dimensional. Also consider $p\vec{Best}_i(t) = \{pBest_{i,1}(t),..., pBest_{i,n}(t)\}$ the best position found by the particular to the time and $g\vec{Best}_i(t) = \{gBest_{i,1}(t),..., gBest_{i,n}(t)\}$ the best position found by the group until the time t .

In standard PSO algorithm, the particles are manipulated according to the following equations:

$$v_{i,n}(t+1) = w * v_{i,n}(t) + c_1 * \phi_1 * (pBest_{i,n}(t) - x_{i,n}(t)) + c_2 * \phi_2 * (gBest_{i,n}(t) - x_{i,n}(t)) \quad (1)$$

$$x_{i,n}(t+1) = x_{i,n}(t) + v_{i,n}(t+1) \quad (2)$$

Where ϕ_1 and ϕ_2 are random numbers uniformly distributed between 0 and 1. The coefficient w determines how much the current influences the previous speed, c_1 and c_2 are constant velocity and acceleration to determine the influence of $p\vec{Best}$ and $g\vec{Best}$ the particle.

2.1 Use of Optimization for Cloud Particle to solve the routing problem

The publication of [8], a model based on cloud optimization for multi-objective particle is presented to solve the routing problem of open vehicles, capable of competing with time windows. In this problem there is competition between the distributors and the goal is to minimize the cost of travel routes and maximize sales while balancing simultaneously obtained the goods distributed among the vehicles. The results were compared with three performance indicators, and show that the proposed approach is efficient to solve the problem with a reasonable computational cost.

The publication of [5], addresses the routing problem capacitated vehicles through a model -based optimization for cloud particles using a probability matrix for encoding and decoding of the particles. The computational results show that the proposed approach is capable of producing excellent results in terms of solution quality when compared with other approaches based on existing PSO, secondly the model requires more computational time in large problems.

In work [1], a model based optimization for Cloud Particles with local search is proposed to solve the routing problem with heterogeneous fleet of vehicles, time windows and simultaneous collection and delivery. The adaptation of this approach to the problem has been studied applied problems in the literature. The results showed that the proposed minor problems when applied to the model was able to obtain quality solutions and in many cases the optimal solutions. For large problems, the model improved the results presented by the authors in previous works in 29 of 56 cases, with an improvement in the quality of the solution around 5.62 %.

In the work of [4] , the Vehicle Routing Problem with Simultaneous Delivery and collection is solved based on a heuristic approach to particle swarm optimization , in which a local search is performed to improve neighborhoods randomly selected solutions during 's search algorithm. Furthermore, the authors implement a strategy called annealing -like to preserve the diversity of the swarm. The computational results show that the proposed algorithm competes with the heuristic approaches the literature and several known solutions improvement in the quality of the solution.

In [7] a model of optimization for cloud particles is used to solve the problem of vehicle routing with stochastic demand. The proposed model uses the local search algorithms 2-opt and 3-opt strategy with path relinking for the improvement of the solution found. The proposed algorithm has been tested in some cases from the literature proposed by [3] for the Vehicle Routing Problem with adjustments in the capacitated demand constraints. The results were considered satisfactory by the authors when compared with other algorithms from the literature for the same problems tested.

In [12] one finds a proposal to solve the routing problem with homogeneous fleet of vehicles, time windows and pickup and delivery, using Cloud Particles. The proposed method works with the particles separated by neighborhoods and adds the information to the neighborhood particle swarm diversify and increase the speed of convergence of the algorithm. Tests conducted by the authors showed that the proposed model is effective because it could reduce the number of vehicles used and the total distance of travel.

3 Proposed algorithm

In this work, the Particle Swarm Optimization for Discrete meta-heuristic is applied to the vehicle routing problem. To apply the model proposed some adjustments should be considered. In this study a particle consists of a vector of order n with students, no schools in vehicles. Each element of the particle is the vehicle used to transport considering its capacity, location and location of the student's school.

A particle is formed by the elements:
- Set students: A_1, A_2, A_3,..., A_n.
- Set schools: E_1, E_2, E_3,..., E_n.
- Set of vehicles: V_1, V_2, V_3,..., V_m.

In this study, particles were only considered feasible. Example particle (Feasible Route):

$$\vec{X}(t) = (V_1, A_1, A_2, A_5, E_2, E_1, E_5, V_2, A_4, A_6, A_9, E_9, E_6, E_4, V_3, A_3, A_7, A_8, A_{10}, E_3, E_7, E_{10}, E_8)$$

In the example above the vehicle 1 (V_1) collecting students 1, 2 and 5 and the leaves in their respective schools, analogously vehicles 2 (V_2) and 3 (V_3) collect students 4, 6, 9, 3.7, 8, 10 respectively and leave them in their schools.

3.1 Changing the position of the elements of the particle

Below is shown the mechanism to make possible changes of position for the elements that make up the particle in order to obtain a reduction in the total mileage of the route. In this paper we began the cloud of particles with only feasible particles.

The process of maintaining a feasible involves three types of particle movement within the vector. The element that changes position in the particle can be a student, a school or a vehicle.

3.1.1 The changed position is a student

The figure 3.1 shows an example of a random draw to determine which element changes position inside the particle with the goal of reducing the total cost of the route (route mileage). To change the student's position within the vector you must ensure that it remains after changing before your school.

Particle	**V_1**	**V_2**	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**E_1**	**E_2**	**E_3**	**E_4**	**E_5**
capacity of the vehicle	0	0	1	2	3	4	5	4	3	2	1	0
Position the student / school / vehicle within the particle	1	2	3	4	5	6	7	8	9	10	11	12
Accumulated mileage (m)	0	0	500	2000	3400	4500	6000	7300	7300	9000	9000	10000

FIGURE 3.1- CHANGED POSITION IS A STUDENT

In Figure 3.1, to make the draw of the element that will change positions within the particle, we have 12 candidates for exchange. 5 If the position is drawn, the element that will change positions the student is 3 (A_3). The student 3 can only change position if you stay in the same vehicle and stay in one of their previous school (E_3) that is occupying position 10 in the particle position.

At this point it is necessary to traverse the elements of the vector representing the particle, to find out what are the limits to make the switch position, ensuring that the change in our position does not make the infeasible particle. The procedure to traverse the vector should be done backwards until the vehicle is found. In the example shown in Figure 3.1 is the pupil in the vehicle A_3 second (V_2). In this case, the A3 can change positions ranging from, and ensuring that the student A_3 appear in the vector after the vehicle 2 (V_2) and before school (E_3).

The capacity control of the vehicle is done by an accountant. In Figure 3.1 the vehicle 1 (V_1) is no passenger, the vehicle 2 (V_2) collects students 1, 2, 3, 4 and 5. Note that the line of control ability for each student collected the vehicle there is an increase in the amount of capacity, and when the vehicle leaves the students in their respective schools E_1, E_2, E_3, E_4 and E_5 accumulated vehicle capacity decreases to zero, which indicates that the vehicle is empty.

Also in Figure 3.1 can be noted that in line with the total mileage accumulated route taken by the vehicle 2 (V_2) is 10.000 meters. The fact that the mileage has remained constant between 1 and 2 schools and between schools 3 and 4 means that 1 and 2 students studying in the same school, similarly for students 3 and 4.

In Figure 3.2, one can observe that the students 1 (A_1) and 2 (A_2) are the same breakpoint and studying in the same school as the accumulated mileage remains constant when the vehicle 2 (V_2) collecting the student 1 (A_1) and then the pupil 2. The same occurs upon landing, the vehicle 2 (V_2) leaves students 1 (A_1) and 2 (A_2) E_1 and E_2 in their schools, note that the school is the same for both students as there was no change in the value of accumulated mileage.

Particle	**V_1**	**V_2**	**A_1**	**A_2**	**E_1**	**E_2**	**A_3**	**E_3**	**A_4**	**E_4**	**A_5**	**E_5**
capacity of the vehicle	0	0	1	2	1	0	1	0	1	0	1	0
Position the student / school / vehicle within the particle	1	2	3	4	5	6	7	8	9	10	11	12
Accumulated mileage (m)	0	0	1000	1000	3000	3000	5000	6500	7000	8000	9000	10000

FIGURE 3.2- CHANGED POSITION IS A STUDENT

In Figure 3.3 the path performed by the vehicle 2 (V2) can be observed.

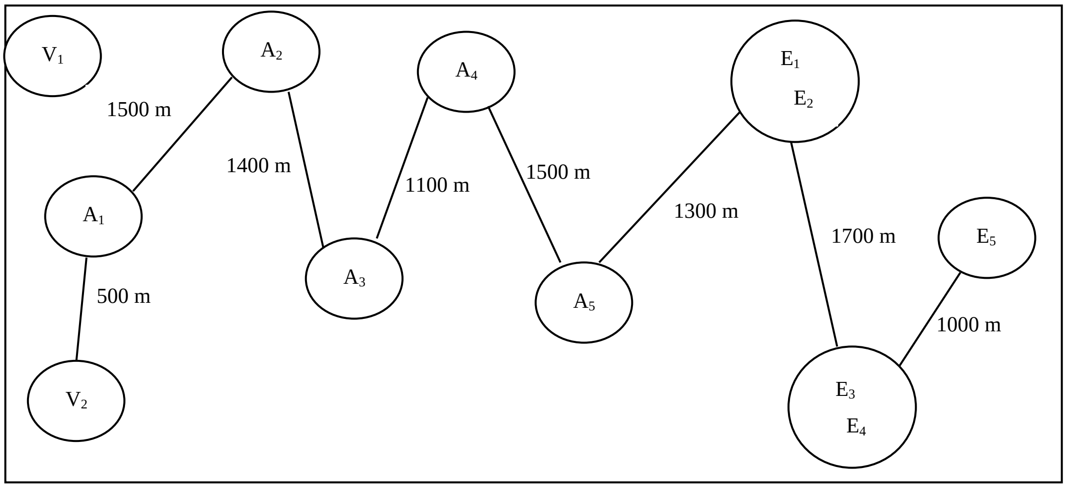

FIGURE 3.3 - PATH TRAVELED IN (V2)

3.1.2 The changed position is a school

In Figure 3.4, the random drawing is held from 12 possible candidates for replacement. If position 10 is drawn, for example, E_4 is the element that must change position. Again cannot occur in the school appear in the vector prior to their student as soon possible options trading are the 11 and 12 positions within the particle. In this case the school E_4 can change positions ranging from $mín = 11$ and $máx = 12$.

Particle	V_1	V_2	A_1	E_1	A_3	A_2	E_2	E_3	A_4	E_4	A_5	E_5
capacity of the vehicle	0	0	1	0	1	0	1	0	1	0	1	0
Position the student / school / vehicle within the	1	2	3	4	5	6	7	8	9	10	11	12

FIGURE 3.4- CHANGED POSITION IS A SCHOOL

To swap the position of the school within the vector you must ensure that it remains after the exchange after the student, otherwise the exchange is not performed.

3.1.3 The changed position is a vehicle

In Figure 3.5, the randomly generated element is a vehicle, for example, the position 2, the vehicle 2 (V_2) to change the position of the particle. In the case of an exchange of vehicle position should be considered that a vehicle cannot occupy a position which separates a student from your school.

Particle	**V_1**	**V_2**	**A_1**	**E_1**	**A_3**	**A_2**	**E_2**	**E_3**	**A_4**	**A_5**	**E_4**	**E_5**
capacity of the vehicle	0	0	1	2	3	4	5	4	3	2	1	0
Position the student / school / vehicle within the	1	2	3	4	5	6	7	8	9	10	11	12

FIGURE 3.5- CHANGED POSITION IS A VEHICLE

Controlling the length of the route is done with a constraint that limits the distance traveled by the vehicle. In this work the threshold value was accumulated mileage of 40 km. Exchanges of position within the particle in the three cases described above are made only if the capacity constraints and control the length of the routearemet.

3.2 Creating Generations

In this work a new proposal for application of meta-heuristic optimization by the Cloud Particle School Transport problem is presented. One of the contributions (innovations) of this work is the creation of new generations of particles in order to increase the diversity of the search for the problem. The proposed algorithm starts with a population of n particles, then a number of iterations is performed. Finally a new population is created, considering feasible particles. This new population has n-1 particles and particle fittness best (gBest) of the previous generation is inserted in the current generation.
The process of creating the generations for a population with p particles can be seen in Figure 3.6.

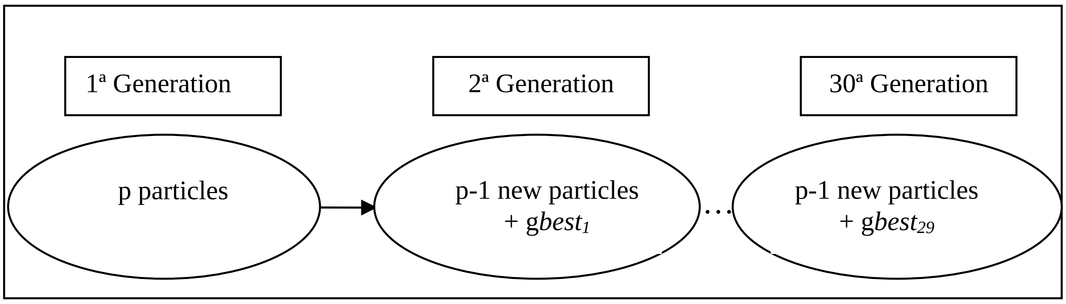

FIGURE 3.6 - CREATION OF GENERATIONS

3.3 Proposed Algorithm

In this section the meta-heuristic approach is described according to Optimization for Cloud Particle Discrete Adapted proposed in this work for the vehicle routing problem with capacitated fixed fleet and heterogeneous simultaneous pickup and delivery. In this section you can observe the innovations proposed in this research, both as regards the codification of the particles as in established operations.

3.3.1 Creation of the particle

The procedure for deploying PSODA the technique proposed in this work is described in the algorithm presented below:
Step 1: start a (matrix) population of particles with positions and velocities in a space of n dimensional problem randomly with uniform distribution;
- In this problem the initial particles were not random, were only considered feasible particles, particles with a given sequence among its elements (vehicle, students and schools), as shown in the example below.

$$\vec{X}(t) = (V_1, A_1, A_2, A_5, E_2, E_1, E_5, V_2, A_4, A_6, A_9, E_9, E_6, E_4, V_3, A_3, A_7, A_8, A_{10}, E_3, E_7, E_{10}, E_8)$$

Step 2: For each particle, evaluate the fitness function (objective function to be minimized-fitness);
- Each particle has its calculated cost (sum of the total distance traveled).

$$\sum_{i,j \in N} c_{i,j} x_{i,j}$$

Where $c_{i,j}$ is the cost (mileage) of the arc that connects vertex i to vertex j enter the route.

Step 3: After generation of the particles:

- Starting position for each particle (feasible route) and speed (list of random transpositions), where the initial position of each particle is equal to the initial pBest of the particle and the number of initial transpositions is equal to the dimension n of the particle.

Step 4: Find the gBest of the population (minimal cost).

Step 5: From the second iteration:

- Compare the assessment of the fitness function of the particle with the pBest of the particle. If the current value (the sum of the mileage) is better than pBest, then the value of pBest shall be equal to the value of the fitness function of the particle, and the pBest location shall be equal to the current location in n dimensional space.

Step 6: Compare the assessment of the fitness function with the previous best fitness value of the population. If current value is better than gBest, gBest update the value for the index and value of current particle.

Step 7: updating the velocity and position of the particle, respectively, according to equations (1) and (2) below:

$$v_{i,n}(t+1) = w * v_{i,n}(t) + c_1 * \phi_1 * (pBest_{i,n}(t) - x_{i,n}(t)) + c_2 * \phi_2 * (gBest_n(t) - x_n(t)) \quad (1)$$

$$x_{i,n}(t+1) = x_{i,n}(t) + v_{i,n}(t+1) \quad (2)$$

- To apply the speed of a particle position, we used the definitions for Discrete PSO presented in [10], with some adaptations unpublished literature. Speed was defined as a list of n random transposition: $v = \{(i_1, j_1), (i_2, j_2), ..., (i_n, j_n)\}$, means the exchange of indices of the vertices i_1 *and* j_1, i_2 *and* j_2, to exchange the vertices i_n and j_n. In this work given a list of transpositions and position of a particle there are three types of exchange:

- If $i < j$, j occupies the position i and i occupies the position $i+1$ in the particle after the exchange;
- If $i = j$ there is no exchange; and
- If $i > j$, i occupies the position j and j occupies the position $j+1$ in the particle after the exchange.

Below is shown an example of the operation of transpositions defined in this work. Given a position and a velocity, trade occurs as shown in the example below.

Are given:

$$v_i(t) = \{(2,3), (1,5), (7,9), (3,5), (1,6), (5,5), (9,7), (8,8), (7,9), (12,12), (4,2), (5,4)\}$$

$$\text{and } x_i(t) = (\overset{1}{V_1}, \overset{2}{A_1}, \overset{3}{A_2}, \overset{4}{E_1}, \overset{5}{E_2}, \overset{6}{V_2}, \overset{7}{A_3}, \overset{8}{A_4}, \overset{9}{A_5}, \overset{10}{E_3}, \overset{11}{E_4}, \overset{12}{E_5})$$

Applying to the list of the position of the particle transpositions, the first transposition is to be done (2,3), the element which occupies position 3 on the particle must occupy the second position, taking the element occupies the second position 3 to the position in the particle. This exchange is said to be feasible, because the student 2 (A_2) remains before school 2 (E_2) after changing position. As the example below shows:

$$x_i(t) = (\overset{1}{V_1}, \overset{2}{A_2}, \overset{3}{A_1}, \overset{4}{E_1}, \overset{5}{E_2}, \overset{6}{V_2}, \overset{7}{A_3}, \overset{8}{A_4}, \overset{9}{A_5}, \overset{10}{E_3}, \overset{11}{E_4}, \overset{12}{E_5})$$

The next transposition is (1,5), if carried out will cause the particle to become infeasible because school occupying the 2 position 5 in particle must move to position 1, which may not occur for thereby it would be before your student (A_2). In this case the exchange is not performed. The next step is to check the third transposition (7, 9), the element that occupies the position 9 (A_5) can switch to position 7, this exchange is possible because the school 5 (E_5) will remain after the student 5 (A_5) performed after the exchange, as the example shows:

$$x_i(t) = (\overset{1}{V_1}, \overset{2}{A_2}, \overset{3}{A_1}, \overset{4}{E_1}, \overset{5}{E_2}, \overset{6}{V_2}, \overset{7}{A_5}, \overset{8}{A_3}, \overset{9}{A_4}, \overset{10}{E_3}, \overset{11}{E_4}, \overset{12}{E_5})$$

The fourth exchange is (3,5), school 2 (E_2) must occupy position 3, the student 1 (A_1) is in position 3 goes to position 4, this exchange is feasible. The change in the particle can be seen below:

$$x_i(t) = (\overset{1}{V_1}, \overset{2}{A_2}, \overset{3}{E_2}, \overset{4}{A_1}, \overset{5}{E_1}, \overset{6}{V_2}, \overset{7}{A_5}, \overset{8}{A_3}, \overset{9}{A_4}, \overset{10}{E_3}, \overset{11}{E_4}, \overset{12}{E_5})$$

The next exchange is (1,6), this exchange can only take place if the capacity constraints and maximum length of the route are met, in which case the vehicle 2 (V_2) is no passenger and vehicle 1 (V_1) gets all passengers.

For transposition (5,5) there is no exchange.

In exchange (9,7) the student 4 (A_4) that occupies the position 9 on the particle must go to the 7 position and pupil 5 (A_5) which is at position 7 is going to position 8, this exchange is feasible.

$$x_i(t) = (\overset{1}{V_1}, \overset{2}{A_2}, \overset{3}{E_2}, \overset{4}{E_1}, \overset{5}{A_1}, \overset{6}{V_2}, \overset{7}{A_4}, \overset{8}{A_5}, \overset{9}{A_3}, \overset{10}{E_3}, \overset{11}{E_4}, \overset{12}{E_5})$$

The following changes are made in the same way.

3 Computational result

The results obtained with the proposed algorithm to the problem of School Transportation in Paraná technique are compared with the results obtained from the application of heuristics ALBH (Adapted Location Based Heuristic) proposed by [9].

In Table 3.1 below the data for each municipality can be observed, considering the number of schools, vehicles available, breakpoints and number of pupils using school transport.

TABLE 3.1 - DATA USED BY PERIOD OF STUDY

Cities	Variables	Morning	Afternoon	Evening	Cities	Variables	Morning	Afternoon	Evening
Abatiá	Vehicles	14	14	14	Mirador	Vehicles	8	8	8
	Students	236	76	56		Students	40	4	12
	Breakpoints	76	21	35		Breakpoints	23	2	8
	Schools	4	5	2		Schools	3	2	2
Agudos do Sul	Vehicles	13	13	13	Moreira Sales	Vehicles	19	19	19
	Students	275	325	41		Students	84	274	90
	Breakpoints	48	51	19		Breakpoints	42	96	41
	Schools	7	5	3		Schools	6	6	3

TABLE 3.1 (CONTINUED) - DATA USED BY PERIOD OF STUDY

Anahy	Vehicles	5	5	5	Nova América da	Vehicles	5	5	5
	Students	19	22	7		Students	57	30	10
	Breakpoints	8	15	6		Breakpoints	17	16	7
	Schools	2	2	1		Schools	2	2	1
Ângulo	Vehicles	4	4	4	Paraíso do Norte	Vehicles	8	8	8
	Students	57	41	7		Students	124	170	70
	Breakpoints	17	12	6		Breakpoints	29	29	19
	Schools	3	3	1		Schools	3	3	2
Ariranha do Ivaí	Vehicles	10	10	10	Paula Freitas	Vehicles	10	10	10
	Students	91	104	44		Students	258	370	95
	Breakpoints	44	48	24		Breakpoints	88	102	42
	Schools	4	3	1		Schools	2	8	2
Bom Jesus do Sul	Vehicles	13	13	13	Pinhais	Vehicles	18	18	18
	Students	137	1	271		Students	113	117	37
	Breakpoints	74	1	81		Breakpoints	29	34	12
	Schools	2	1	5		Schools	18	14	3
Cafeara	Vehicles	6	6	6	Prado Ferreira	Vehicles	4	4	4
	Students	9	53	44		Students	45	12	18
	Breakpoints	6	21	10		Breakpoints	12	3	6
	Schools	2	2	2		Schools	2	1	1
Céu Azul	Vehicles	17	17	17	Rio Negro	Vehicles	28	28	28
	Students	147	397	43		Students	353	401	67
	Breakpoints	44	78	21		Breakpoints	52	52	19
	Schools	6	7	2		Schools	13	15	3
Diamante do Oeste	Vehicles	9	9	X	Santa Amélia	Vehicles	5	5	5
	Students	262	279	X		Students	174	63	40
	Breakpoints	6	50	X		Breakpoints	31	17	18
	Schools	2	2	X		Schools	5	4	2
Fernandes Pinheiro	Vehicles	11	11	11	Santa Mariana	Vehicles	12	12	12
	Students	599	179	41		Students	33	76	50
	Breakpoints	95	60	17		Breakpoints	14	21	12
	Schools	6	4	1		Schools	6	5	4
Figueira	Vehicles	11	11	11	São Sebastião da	Vehicles	5	5	5
	Students	422	331	101		Students	118	33	9
	Breakpoints	49	28	20		Breakpoints	28	10	4
	Schools	8	8	3		Schools	5	4	2
Flor da Serra do Sul	Vehicles	14	14	14	Teixeira Soares	Vehicles	14	14	14
	Students	386	230	9		Students	488	613	87
	Breakpoints	109	57	6		Breakpoints	129	21	50
	Schools	4	5	1		Schools	9	5	2
Iracema do Oeste	Vehicles	4	4	4	Tunas do Paraná	Vehicles	8	8	8
	Students	47	1	4		Students	204	259	124
	Breakpoints	22	1	4		Breakpoints	26	26	33

TABLE 3.1 (CONTINUED) - DATA USED BY PERIOD OF STUDY

Schools	3	1	1		Schools	2	3	2
Vehicles	13	13	13	Uniflor	Vehicles	5	5	5
Students	233	139	21		Students	40	51	20
Breakpoints	75	19	11		Breakpoints	19	17	11
Schools	10	9	3		Schools	2	2	2
Vehicles	4	X	4	Uraí	Vehicles	20	20	20
Students	59	X	1		Students	235	226	94
Breakpoints	27	X	1		Breakpoints	65	64	25
Schools	5	X	1		Schools	7	7	3
Vehicles	9	9	9	Ventania	Vehicles	18	18	18
Students	179	175	10		Students	214	359	101
Breakpoints	67	55	9		Breakpoints	33	66	28
Schools	6	6	1		Schools	5	6	2

Table 3.2 shows the results obtained from the application of the proposed methodology for 32 cities considering the sum of mileage routes to and from the three study periods: morning, afternoon and evening.

TABLE 3.2 - COMPARISON BETWEEN OF THE TECHNIQUE PSODA AND ALBH FOR 32 CITIES IN PARANÁ

Cities	Technique	Total Daily Mileage (Km)	absolute error	relative error	Economy daily (%)
Abatiá	ALBH	1.285,386	17,886	0,0139	1,39
	PSODA	1.267,500			
Agudos do Sul	ALBH	1.037,460	78,974	0,0761	7,61
	PSODA	958,486			
Anahy	ALBH	219,626	32,362	0,1474	14,74
	PSODA	187,264			
Ângulo	ALBH	351,162	45,772	0,1303	13,03
	PSODA	305,390			
Ariranha do Ivaí	ALBH	764,426	10,760	0,0141	1,41
	PSODA	753,666			
Bom J. do Sul	ALBH	1.050,968	-5,413	-0,0052	-0,52
	PSODA	1.056,381			
Cafeara	ALBH	404,730	26,562	0,0656	6,56
	PSODA	378,168			
Céu Azul	ALBH	1.698,998	22,656	0,0133	1,33
	PSODA	1.676,342			

TABLE 3.2 (CONTINUED) - COMPARISON BETWEEN OF THE TECHNIQUE PSODA AND ALBH FOR 32 CITIES IN PARANÁ

Diamante do Oeste	ALBH	524,942	3,460	0,0066	0,66
	PSODA	521,482			
Fernandes Pinheiro	ALBH	1.875,266	59,260	0,0316	3,16
	PSODA	1.816,006			
Figueira	ALBH	1.155,094	1,196	0,0010	0,10
	PSODA	1.153,898			
Flor da S. do Sul	ALBH	1.824,160	79,754	0,0437	4,37
	PSODA	1.744,406			
Iracema do Oeste	ALBH	217,740	25,948	0,1192	11,92
	PSODA	191,792			
Loanda	ALBH	1.235,196	57,256	0,0464	4,64
	PSODA	1.177,940			
Lobato	ALBH	214,694	1,020	0,0048	0,48
	PSODA	213,674			
Marilena	ALBH	1.108,276	12,490	0,0113	1,13
	PSODA	1.095,786			
Mirador	ALBH	338,620	72,348	0,2137	21,37
	PSODA	266,272			
Moreira Sales	ALBH	1.606,714	55,970	0,0348	3,48
	PSODA	1.550,744			
Nova A. da Colina	ALBH	374,626	26,408	0,0705	7,05
	PSODA	348,218			
Paraíso do Norte	ALBH	618,304	-3,438	-0,0056	-0,56
	PSODA	621,742			
Paula Freitas	ALBH	1.941,674	54,612	0,0281	2,81
	PSODA	1.887,062			
Pinhais	ALBH	677,014	22,666	0,0335	3,35
	PSODA	654,346			
Prado Ferreira	ALBH	149,378	-0,996	-0,0067	-0,67
	PSODA	150,374			
Rio Negro	ALBH	1.004,848	17,748	0,0177	1,77
	PSODA	987,100			
Santa Amélia	ALBH	792,368	68,608	0,0866	8,66
	PSODA	723,760			

TABLE 3.2 (CONTINUED) - COMPARISON BETWEEN OF THE TECHNIQUE PSODA AND ALBH FOR 32 CITIES IN PARANÁ

Santa Mariana	ALBH	597,318	23,906	0,0400	4,00
	PSODA	573,412			
São S. da Amoreira	ALBH	508,358	79,384	0,1562	15,62
	PSODA	428,974			
Teixeira Soares	ALBH	3.474,762	50,040	0,0144	1,44
	PSODA	3.424,722			
Tunas do Paraná	ALBH	783,222	27,316	0,0349	3,49
	PSODA	755,906			
Uniflor	ALBH	430,768	60,422	0,1403	14,03
	PSODA	370,346			
Uraí	ALBH	1.386,016	25,494	0,0184	1,84
	PSODA	1.360,522			
Ventania	ALBH	1.517,004	73,672	0,0486	4,86
	PSODA	1.443,332			
Total	ALBH	31.169,11	1.124,1	0,0514	5,14
	PSODA	30.045,01			

The data in this table show that the application of the methodology presented in this work provided savings for these municipalities ranging between 0,10% and 21,37% in the total daily mileage. This study considered whether the path of return. Of the 32 cities tested decreased the total daily mileage traveled in 29 municipalities, only 3 municipalities PSODA technique was surpassed by ALBH. In Table 3.2 it is observed that, using the technique ALBH total daily mileage for 32 municipalities was 31169,116 Km and using the technique proposed in this paper PSODA the total daily mileage was 30045,013 Km, the difference is 1.124, 103 km daily.

4 Conclusions

In this paper a methodology is presented to solve the routing problem to School Transport thirty-two cities in the state of Paraná using the optimization algorithm for cloud particles. The real case discussed in this work and available database provide an opportunity to develop a methodology capable of contributing to formatting and alternative practices for the transport implementations, and specifically school transportation.

The computational experiment conducted showed that the proposed algorithm could obtain good quality solutions. The results indicate the effectiveness of the proposed approach to problem sets analyzed.

The routing of transport is a very important aspect, because it ends up influencing factors involved in the planning of the service, since an efficient routing can minimize travel time, optimize the occupancy of vehicles and improve the existing service, which will ultimately reduce the cost with the system and improving its quality .

Finally, the results clearly show the potential of the presented approach, where lower cost solutions are obtained for relatively large problems in times of low processing.

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